PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture - 12

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Topics *to be covered*

- **A** Logarithmic Inequalities
- **B** Introduction to Modulus
- **C** Problem Practice

Homework Discussion

QUESTION

Tah 03

Solve the following logarithmic equations:

1. $\log_3(x^2-3x-5)=\log_3(7-2x)$
2. $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$
3. $25^{\log_{10}x}=5+4\times\log_{10}5$
4. $1+2\log_{x+2}5=\log_5(x+2)$
5. $2\log_2(\log_2 x)+\log_{\frac{1}{2}}(\log_2(2\sqrt{2}x))=1$

Solve $25^{\log_{10}x}=5+4x^{\log_{10}5}$

TAH -03 ka 3rd Question ye hai usmein typing error hai

$\log_2(\log_2 x)^2-\log_2(\log_2 2\sqrt{2}+\log_2 x)=1$

let $\log_2 x=t$ $\quad \log_2 t^2-\log_2(3/2+t)=1$

$$\log_2\left(\frac{t^2}{\frac{3}{2}+t}\right)=1$$

$$\frac{t^2}{\frac{3}{2}+t}=2^1$$

$$t^2=3+2t$$

$$t^2-2t-3=0$$

$$(t-3)\,(t+1)=0$$

$$t=3,-1$$

$$\log_2 x=3 \text{ or } \log_2 x=-1$$

$$x=8,\ \cancel{\tfrac{1}{2}}$$

QUESTION

Solve the following equations :

(vi) $\log_{5-x}(x^2 - 2x + 65) = 2$

(vii) $\log_{10} 5 + \log_{10}(x + 10) - 1 = \log_{10}(21x - 20) - \log_{10}(2x - 1)$

(viii) $x^{1+\log_{10} x} = 10x$

(ix) $2\left(\log_x \sqrt{5}\right)^2 - 3\log_x \sqrt{5} + 1 = 0$

(x) $3 + 2\log_{x+1} 3 = 2\log_3(x + 1)$

$\log_{10}5(x+10) - \log_{10}10 = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$\log_{10}\left(\frac{x+10}{2}\right) = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$\frac{x+10}{2} = \frac{21x-20}{2x-1}$

Answers:

i.	$\{1+\sqrt{3}\}$	ii.	$\{3\}$	iii.	$\{4\}$	iv.	$\{2\}$
v.	$\{0\}$	vi.	$\{-5\}$	vii.	$\{3/2, 10\}$	viii.	$\{10^{-1}, 10\}$
ix.	$\{\sqrt{5}, 5\}$	x.	$\{-(3-\sqrt{3})/3, 8\}$				

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

Logarithmic Inequalities

If f is an increasing fn

$x > y \iff f(x) > f(y)$

Kisi inequality mai Inc fn laagane Yaa hataane pe sign of Inequality mai koi change nahi hotaa

Ex: $f(x) = e^x$ is an inc fn

find range of x: $e^{x^2+2} \geq e^4$

$x^2 + 2 \geq 4$

$x^2 - 2 \geq 0$

$(x - \sqrt{2})(x + \sqrt{2}) \geq 0$

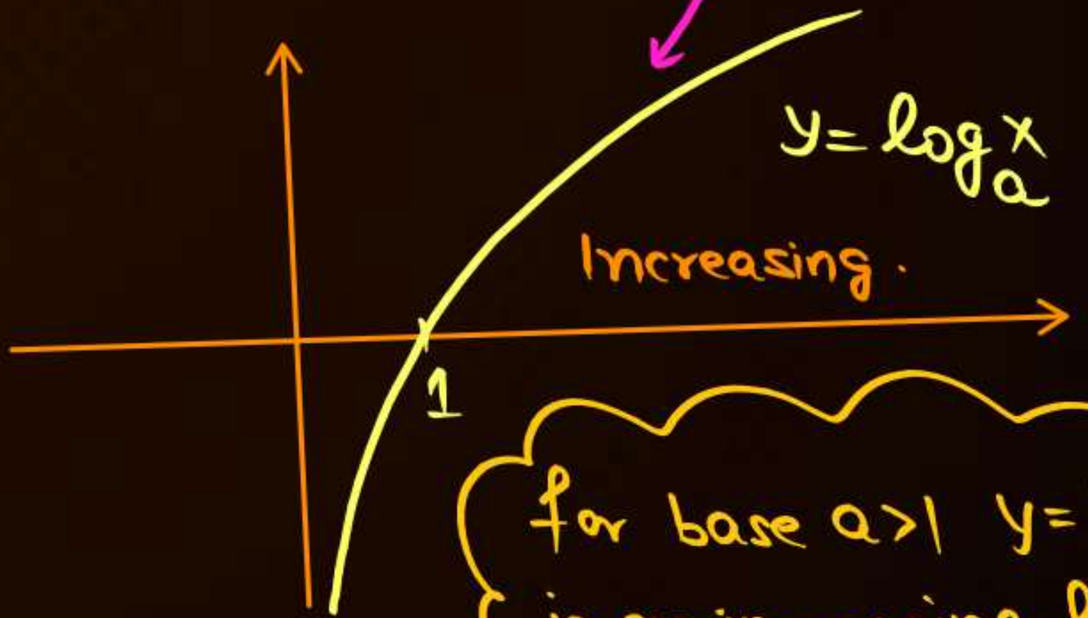
$+ \quad - \quad +$

$-\sqrt{2} \quad \sqrt{2}$

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

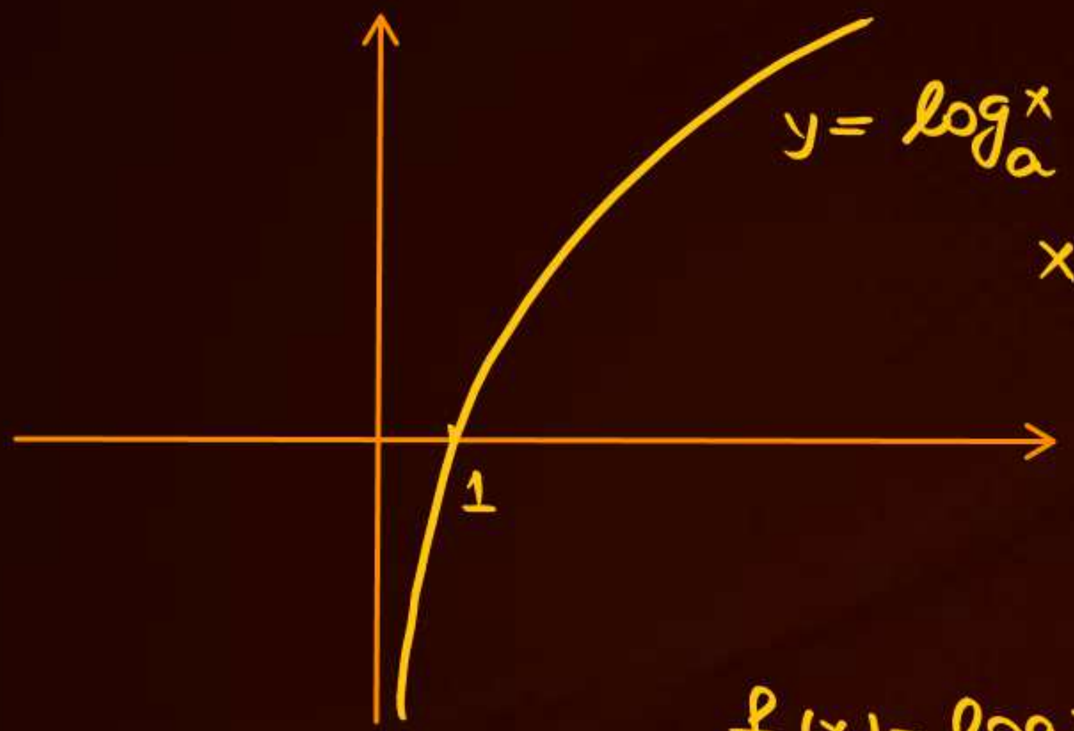
$y = \log_2 x$

x	y
1	0
$\frac{1}{2}$	−1
$\frac{1}{4}$	−2
2	1
4	2



$y = \log_a x, \ a > 1$

for base $a > 1$ $y = \log_a x$ is an increasing function



$y = \log_a x \quad , a > 1$

$x \in R^+, \quad y \in (-\infty, \infty)$

$f(x) = \log_a x \; , a > 1 \quad$, is inc fn.

Domain $= R^+$

Range $= R$

Ex: $\log_{10} x \geq 2$

inc fn.

lallu: $\log_{10} x \geq \log_{10} 100$

$x \geq 100$

kallu

$\log_{10} x \geq 2$

$x \geq 10^2 = 100$

Ex: $\log_{10}(x-1) < 1$

$x - 1 < 10^1 \;\&\; x - 1 > 0$

$x < 11 \;\&\; x > 1$

$x \in (1, 11)$

QUESTION

$$\log_7\left(\frac{2x-6}{2x-1}\right) > 0$$

$\log_a x$ is defined only if $x \in R^+$, $a>0, a\neq 1$

$\frac{2x-6}{2x-1} > 7^0$ & $\frac{2x-6}{2x-1} > 0$

No Need

$\frac{2x-6}{2x-1} > 1$

$\frac{2x-6}{2x-1} - 1 > 0$

$\frac{\cancel{2x}-6-\cancel{2x}+1}{2x-1} > 0$

$\frac{-5}{2x-1} > 0$

$\frac{1}{2x-1} < 0$ → $2x-1<0$

$x < 1/2 \Rightarrow$ Ans: $x \in (-\infty, 1/2)$

QUESTION

$\underbrace{\log_5}_{\text{Inc}}(x^2 - 5x + 6) > -\mathbf{1}$

$x^2 - 5x + 6 > 5^{-1}$ & $x^2 - 5x + 6 > 0$

$\downarrow$ (No Need)

$x^2 - 5x + 6 > \frac{1}{5}$

$5x^2 - 25x + 30 > 1$

$5x^2 - 25x + 29 > 0$

$x = \frac{25 \pm \sqrt{625 - 580}}{10}$

$x = \frac{25 \pm \sqrt{45}}{10}$

$x = \frac{25 \pm 3\sqrt{5}}{10}$

$5\cdot\left(x - \frac{25-3\sqrt{5}}{10}\right)\left(x - \frac{25+3\sqrt{5}}{10}\right) > 0$

$+ \quad - \quad +$

$\frac{25-3\sqrt{5}}{10} \qquad \frac{25+3\sqrt{5}}{10}$

$x \in \left(-\infty, \frac{25-3\sqrt{5}}{10}\right) \cup \left(\frac{25+3\sqrt{5}}{10}, \infty\right)$

QUESTION

$\log_2(\log_3(\log_5 x)) > 0$

$\log_3(\log_5 x) > 2^0$

$\log_3(\log_5 x) > 1$

$\log_5 x > 3^1 = 3$

$x > 5^3$

$x > 125$ Ans.

& $\log_3(\log_5 x) > 0$ $\downarrow$ (No Need)

& $\log_5 x > 0$ $\downarrow$ (No Need)

& $x > 0$ $\downarrow$ (No Need)

QUESTION [WB JEE 2022]

If x satisfies the inequality $\log_{25} x^2 + (\log_5 x)^2 < 2$, then x belongs to

(A) $\left(\frac{1}{5}, 5\right)$

(B) $\left(\frac{1}{25}, 5\right)$

(C) $\left(\frac{1}{5}, 25\right)$

(D) $\left(\frac{1}{25}, 25\right)$

$\log_{5^2} x^2 + (\log_5 x)^2 < 2$

$\frac{2}{2}\log_5|x| + (\log_5 x)^2 < 2$

(clearly $x > 0$)

$\log_5 x + (\log_5 x)^2 < 2$

let $\log_5 x = t$

$t^2 + t - 2 < 0$

$(t+2)(t-1) < 0$

$t \in (-2, 1)$

$-2 < t < 1 \Rightarrow -2 < \log_5 x < 1$

$5^{-2} < x < 5^1$

$x \in (1/25, 5)$

Ans. B

If f is a dec. fn then

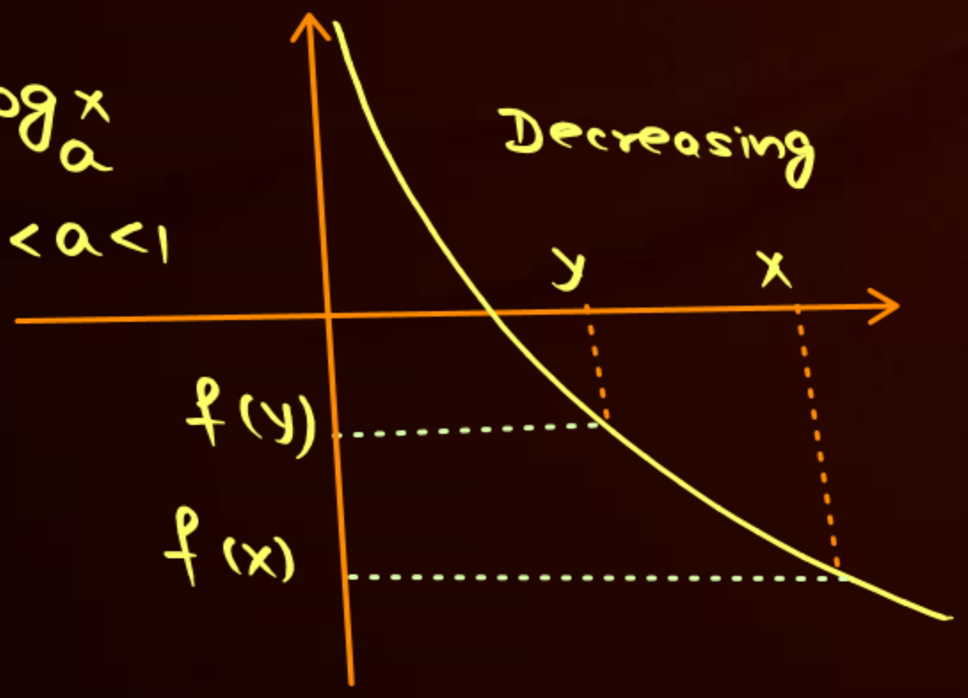
$x > y \iff f(x) < f(y)$

Kisi Inequality mai dono side
Koi dec. fn lagao yaa hatao
sign of inequality reverse ho jaata hai

$y = \log_a x \; , \; x \in R^+ ; \; 0 < a < 1$ is decreasing fn

$y = \log_a x$

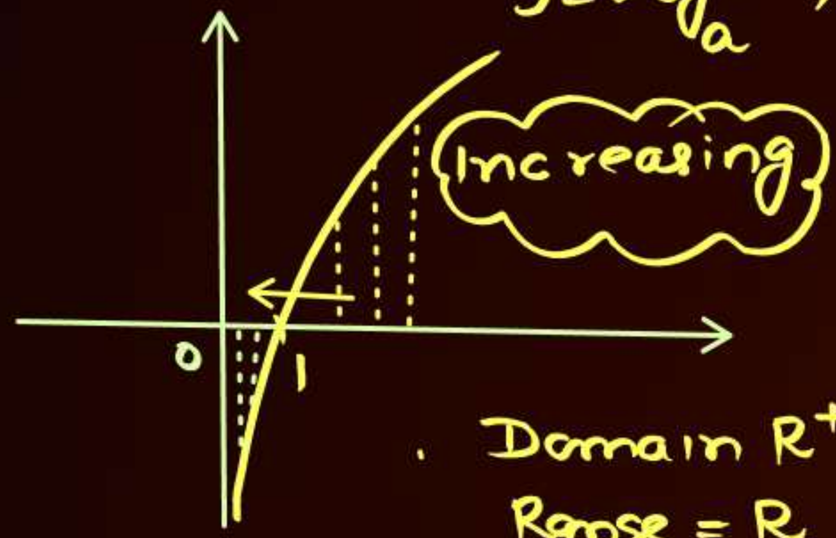
$0 < a < 1$



Domain R^+

Range : R

$y = \log_a x$, $x \in R^+$, $a > 1$



Domain R^+

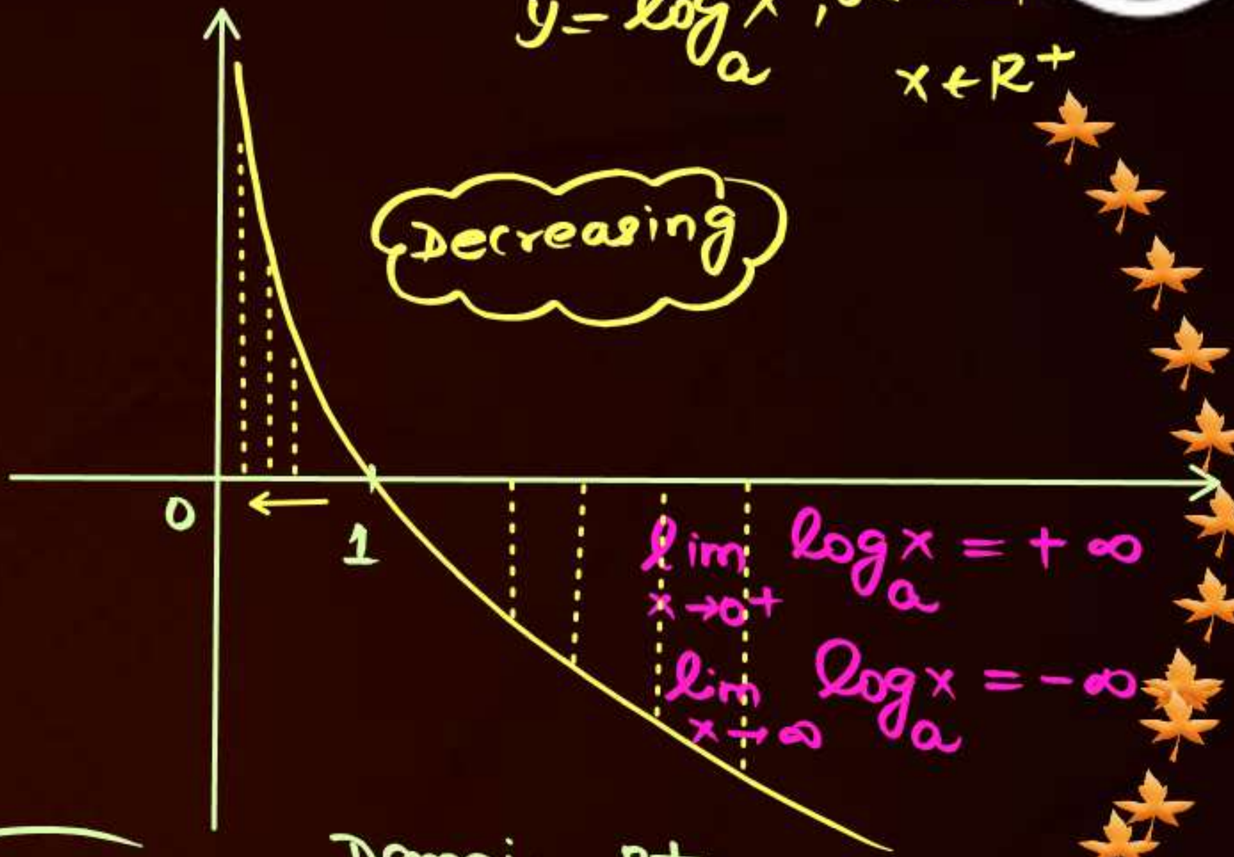
Range $= R$

$\lim_{x \to 0^+} \log_a x = -\infty$

$\lim_{x \to \infty} \log_a x = \infty$

$y = \log_a x$, $0 < a < 1$

$x \in R^+$



$\lim_{x \to 0^+} \log_a x = +\infty$

$\lim_{x \to \infty} \log_a x = -\infty$

Domain $= R^+$

Range $= R$

$f(x) = \log_a x$ — Domain R^+

— Range R .

$y = \log_2 x \qquad y = \log_3 x \quad , y = \log_4 x$

$\log_4 \frac{1}{8} = \log_{2^2} 2^{-3}$

$= -\frac{3}{2}$

$\log_3 \frac{1}{8} = -3 \log_3 2.$

$3^{-2} = \frac{1}{9} < \frac{1}{8}$

$\log_3 1/8 > -2$

$\log_a x$ kaa value
voh power hoti hai
jo 'a' pe lagayi
jaaye taaki x aa jayay

QUESTION [WB JEE 2016]

If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(A) $(2, \infty)$

(B) $(1, 2)$

(C) $(-2, -1)$

(D) None of these

$\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\log_{0.3}(x-1) < \frac{1}{2}\log_{0.3}(x-1)$

$2\log_{0.3}(x-1) < \log_{0.3}(x-1)$

$\log_{0.3}(x-1)^2 < \log_{0.3}(x-1) \Rightarrow (x-1)^2 > x-1$

decreasing fn

$(x-1)^2 - (x-1) > 0$

$(x-1)(x-2) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

$x - 1 > 0$

$x > 1$

$\cap$

$x \in (2, \infty)$

Ans. A

QUESTION

$$\log_{0.2}(x^2 - x - 2) > \log_{0.2}(-x^2 + 2x + 3)$$

$x^2 - x - 2 < -x^2 + 2x + 3$

$2x^2 - 3x - 5 < 0$

$2x^2 - 5x + 2x - 5 < 0$

$(2x - 5)(x + 1) < 0$

$x \in (-1, 5/2)$

& $x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$x \in (-\infty, -1) \cup (2, \infty)$

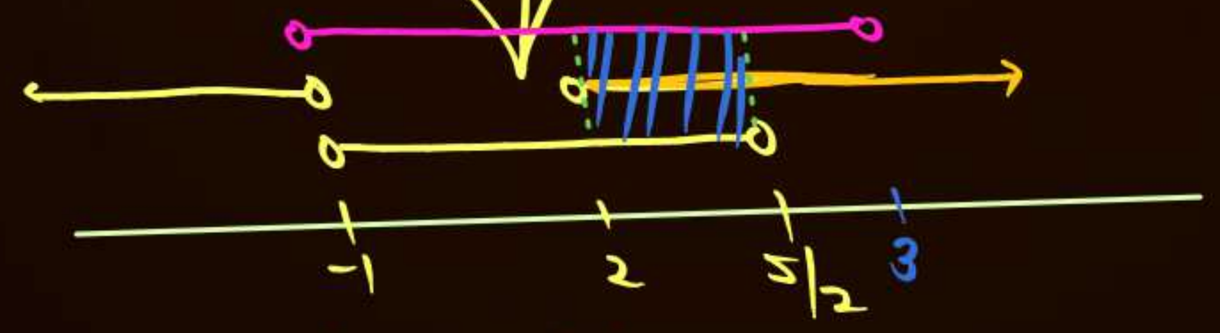
& $-x^2 + 2x + 3 > 0$

$x^2 - 2x - 3 < 0$

$(x-3)(x+1) < 0$

$x \in (-1, 3)$

$\cap$



$x \in (2, 5/2)$ Ans

QUESTION

If $\log_{0.5}\left(\log_5(x^2-4)\right) > \log_{0.5} 1$ then find possible values of x

$\log_5(x^2-4) < 1$

$x^2-4 < 5$

$x^2-9 < 0$

$(x-3)(x+3) < 0$

$x \in (-3, 3)$

& $\log_5(x^2-4) > 0$

$x^2-4 > 5^0$

$x^2-5 > 0$

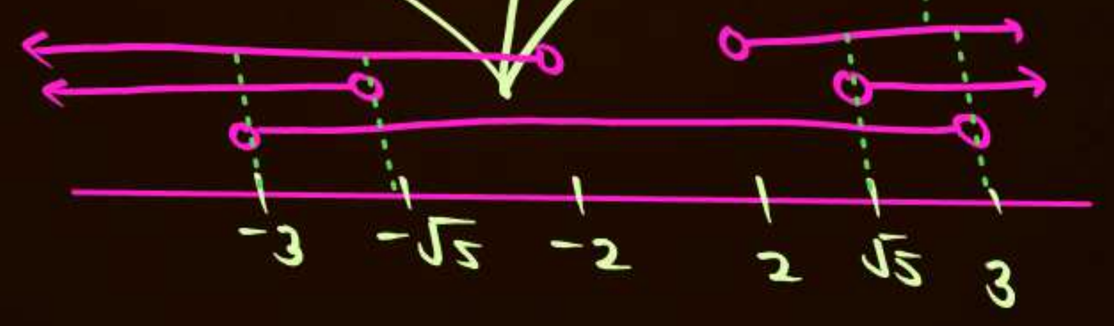
$x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

& $x^2-4 > 0$

$(x-2)(x+2) > 0$

$x \in (-\infty, -2) \cup (2, \infty)$

$\cap$



$x \in (-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$

Variable Base

$$\log_{f(x)} g(x) > \log_{f(x)} h(x)$$

$g(x) > 0$ & $h(x) > 0$

$\cap$

B

case (I) if $f(x) > 1 \Rightarrow$

$g(x) > h(x)$

$\cap$

case (I) Ans

case (II) if $0 < f(x) < 1 \Rightarrow$

$g(x) < h(x)$

$\cap$

case (II) Ans

UNION

A

Cases ke Answers kaa hamesha union hotaa hai

Final Ans: $A \cap B$

QUESTION

Also $2x - \frac{3}{4} > 0 \Rightarrow x > \frac{3}{8}$ — (A)

$$\log_x\left(2x - \frac{3}{4}\right) > 2$$

Case (i) if $x > 1$

$2x - \frac{3}{4} > x^2$

$8x - 3 > 4x^2$

$4x^2 - 8x + 3 < 0$

$4x^2 - 6x - 2x + 3 < 0$

$(2x-1)(2x-3) < 0$

$\frac{1}{2} < x < 3/2$

$x \in (1/2, 3/2)$

$\cap$

$x \in (1, 3/2)$

Case (ii) if $0 < x < 1$

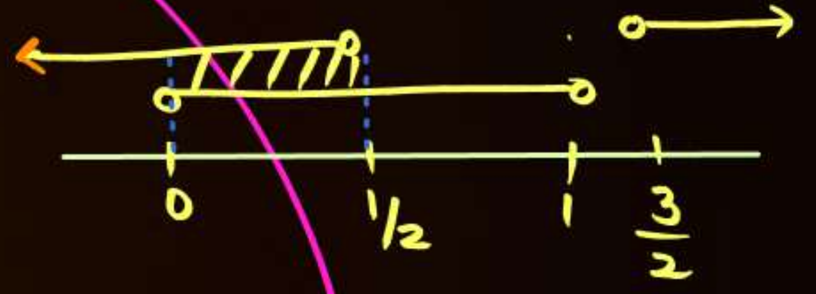
$2x - \frac{3}{4} < x^2$

$8x - 3 < 4x^2$

$4x^2 - 8x + 3 > 0$

$(2x-1)(2x-3) > 0$

$x \in (-\infty, 1/2) \cup (3/2, \infty)$



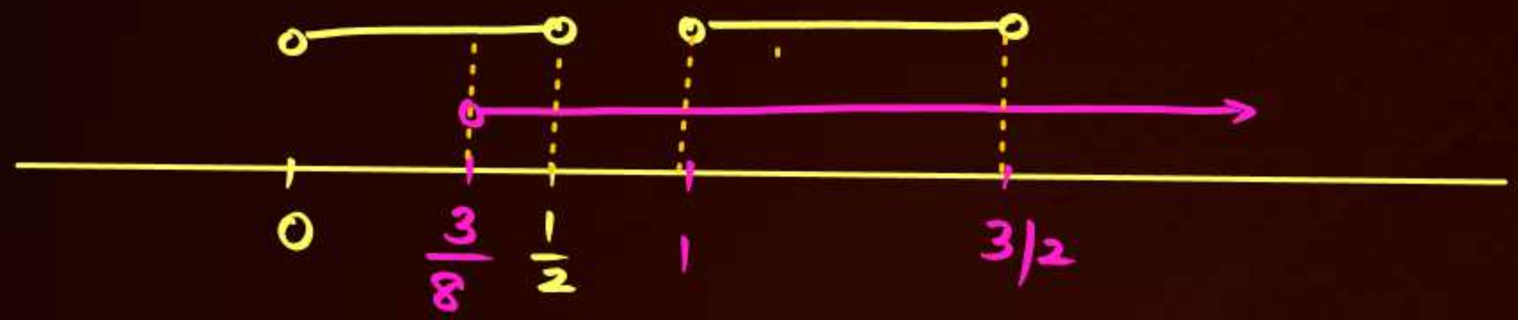
$\cap$

$x \in (0, 1/2)$

UNION ($\cup$)

$x \in (0, 1/2) \cup (1, 3/2)$ — (B)

$\underbrace{A \cap B}$



Ans: $x \in (3/8, 1/2) \cup (1, 3/2)$ Ans

QUESTION

$$\log_{2x+3}(x^2) < \log_{2x+3}(2x+3)$$

$x^2 > 0$

$\downarrow$

$x \in R_0 = (-\infty, \infty) - \{0\}$ — (A)

$$\log_{2x+3} x^2 < 1$$

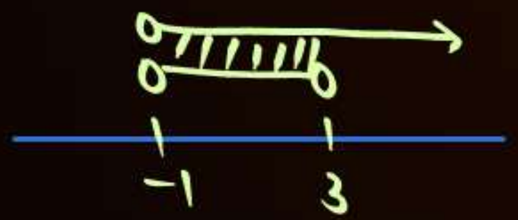
Case (I) If $2x+3 > 1 \Rightarrow x > -1$

$x^2 < 2x+3$

$x^2 - 2x - 3 < 0$

$(x-3)(x+1) < 0$

$x \in (-1, 3)$ $\cap$



$x \in (-1, 3)$

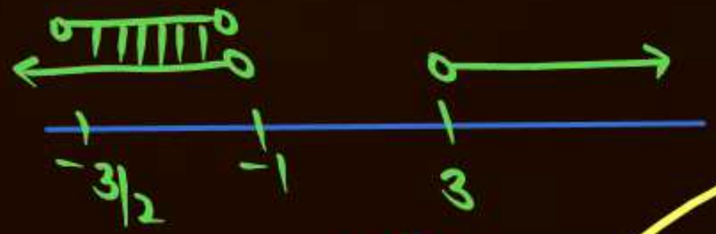
Case (II) If $0 < 2x+3 < 1 \Rightarrow -3 < 2x < -2$

$-\frac{3}{2} < x < -1$

$x^2 > 2x+3$

$x^2 - 2x - 3 > 0 \Rightarrow (x+1)(x-3) > 0$

$x \in (-\infty, -1) \cup (3, \infty)$ $\cap$



$x \in (-3/2, -1)$

UNION

$x \in (-3/2, -1) \cup (-1, 3)$ — (B)

Ans: $A \cap B$

$$x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, 3)$$

OR

$$x \in \left(-\frac{3}{2}, 3\right) - \{0, -1\}$$

QUESTION [JEE Mains 2023]

Tah 01

The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

(A) 8

(B) 7

(C) 5

(D) 6

QUESTION

$\log_{x+3}(x^2 - x) < 1$ Tah 02

Saari Class Illustrations
Retry karni Hai

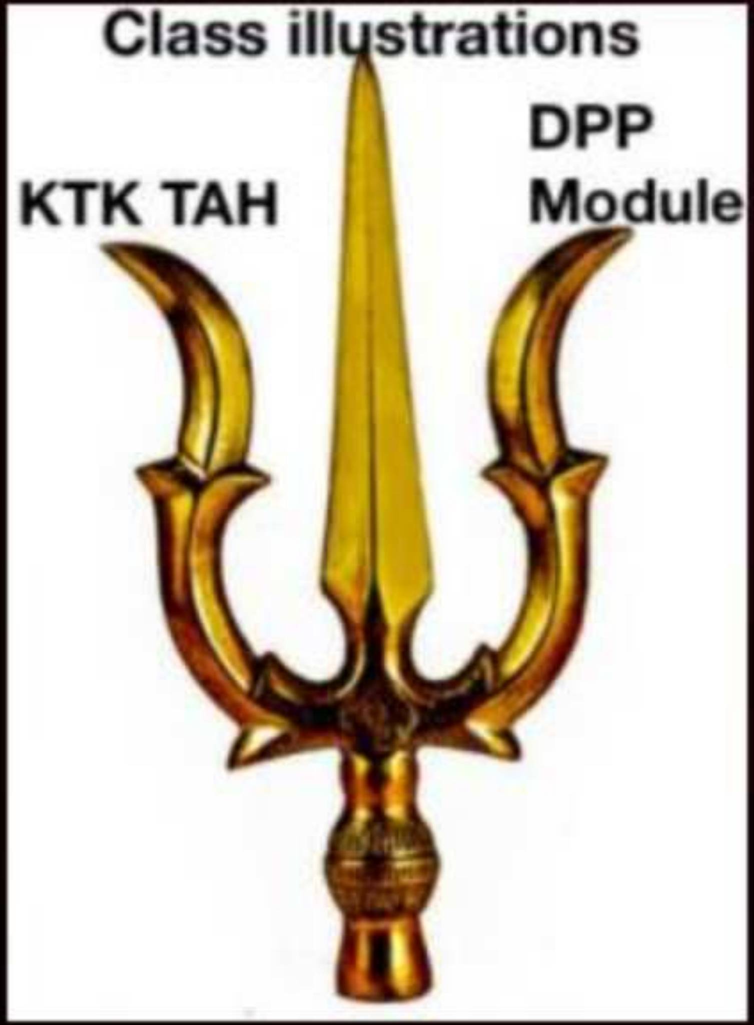
Home Challenge-04

Let a, b, c be three distinct positive real numbers such that

$(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then, $6a + 5bc$ is equal to

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank



QUESTION [MHT CET 2023 (9 May)]

(KTK 01)

If $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$ and $x = 2^k$ then k is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{8}$

Ans. C

QUESTION [BITSAT 2021]

(KTK 02)

If $\log_7 5 = a$, $\log_5 3 = b$ and $\log_3 2 = c$, then the logarithm of the number 70 to the base 225 is

(A) $\dfrac{1 - a + abc}{2a(1 + b)}$

(B) $\dfrac{1 - a - abc}{2a(1 + b)}$

(C) $\dfrac{1 + a - abc}{2a(1 + b)}$

(D) $\dfrac{1 + a + abc}{2a(1 + b)}$

Ans. D

QUESTION [BITSAT 2020]

(KTK 03)

If $\log_5 \frac{(a+b)}{3} = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4+b^4}{a^2b^2}$ is equal to

(A) 50

(B) 47

(C) 44

(D) 53

Ans. B

QUESTION [WB JEE 2024]

(KTK 04)

If $(x^2 \log_x 27) \cdot \log_9 x = x + 4$ then the value of x is

(A) 2

(B) $-\frac{4}{3}$

(C) -2

(D) $\frac{4}{3}$

Ans. A

QUESTION [WB JEE 2020]

(KTK 05)

If $2\log(x+1) - \log(x^2 - 1) = \log 2$, then $x =$

(A) only 3

(B) -1 and 3

(C) only -1

(D) 1 and 3

Ans. A

QUESTION [WB JEE 2018]

(KTK 06)

If $x + \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6$, then the value of x is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 1

(D) 2

Ans. C

QUESTION [WB JEE 2019]

(KTK 07)

If $\log_2 6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$, then the value of x are

(A) $\frac{1}{4}, \frac{1}{3}$

(B) $\frac{1}{4}, \frac{1}{2}$

(C) $-\frac{1}{4}, \frac{1}{2}$

(D) $\frac{1}{3}, -\frac{1}{2}$

Ans. B

QUESTION [WB JEE 2017]

(KTK 08)

If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(A) 125

(B) 25

(C) 513

(D) 243

Ans. A

QUESTION [COMEDK 2023]

(KTK 09)

The value of $a^{\log_b c} - c^{\log_b a}$, where $a, b, c > 0$ but $a, b, c \neq 1$, is

(A) a

(B) b

(C) c

(D) 0

Ans. D

QUESTION [COMEDK 2022]

(KTK 10)

The value of $3^{\log_4 5} - 5^{\log_4 3}$ is

A 0

B 1

C 2

D 4

Ans. A

QUESTION [COMEDK 2021]

(KTK 11)

$8^{3\log_8 5}$ is equal to

(A) $\log_8 25$

(B) 120

(C) 125

(D) $\log_8 15$

Ans. C

QUESTION [COMEDK 2020]

(KTK 12)

$7^{2\log_7 5}$ is equal to

(A) 5

(B) $\log_7 35$

(C) $\log_7 25$

(D) 25

Ans. D

QUESTION

(KTK 13)

Find the exhaustive solutions set of $\frac{(x^2-9)^{101}(x^2+6)(x^2-4)^{100}}{(x^2-5x+6)^{13}(x^2-16)^{16}} > 0.$

Ans. $(-\infty, -3) \cup (2, \infty) - \{\pm 4, 3\}$

QUESTION

(KTK 14)

Find the exhaustive solutions set of $\dfrac{(x-4)^{30}(x^2-9)^9(x^2-3x+2)^{17}(3x^2+10)^{10}}{(x^2-5x+6)^{52}(x^2-25)^{60}(x^2+10)^{11}} \leq 0.$

Ans. $[-3,1] \cup (2,3)$

QUESTION

(KTK 15)

Solve in real numbers the equation $\sqrt{x}+\sqrt{y}+2\sqrt{z-2}+\sqrt{u}+\sqrt{v}=x+y+z+u+v$.

Ans. $x=y=u=v=1/4, z=3$

QUESTION

(KTK 16)

Find all pair of positive integer (m, n) that satisfy $mn + 3m - 8n = 59$.

Ans. 3

QUESTION

(KTK 17)

The least value of the expression $(x+y)(y+z)$ where given that $x, y, z > 0$ and $xyz(x+y+z)=1$

Ans. 2

Today's BPP

QUESTION

Solve the following inequalities:

(a) $\log(x^2 - 2x - 2) \le 0$

(b) $\log_5(x^2 - 11x + 43) < 2$

(c) $2 - \log_2(x^2 + 3x) \ge 0$

(d) $\log_{1.5}\frac{2x-8}{x-2} < 0$

(e) $\log_3\frac{1+2x}{1+x} < 1$

(f) $\log_4\frac{3x+2}{x} \le 0.5$

(g) $\log_2\frac{x^2-4x+2}{x+1} \le 1$

(h) $\log_2^2\left(\frac{4x-3}{4-3x}\right) > -\frac{1}{2}$

Answers:

(a) $[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$;

(b) $(2,9)$;

(c) $[-4,-3) \cup (0,1]$;

(d) $(4,6)$;

(e) $(-\infty,-2) \cup (-1/2,\infty)$;

(f) $[-2,-2/3)$;

(g) $[0, 2-\sqrt{2}) \cup (2+\sqrt{2}, 6]$;

(h) $\left(\frac{3}{4},\frac{4}{3}\right)$

Homework From Module

Prarambh (Topicwise)	: Q1 to Q17
Prabal (JEE Main Level)	: Q1 to Q7

Solution to Previous TAH

QUESTION

If $\log_{10} 5 = \mathrm{a}$ and $\log_{10} 3 = \mathrm{b}$, then :

A $\log_{30} 8 = \dfrac{3(1+\mathrm{a})}{\mathrm{b}+1}$

B $\log_{30} 8 = \dfrac{3(1-\mathrm{a})}{\mathrm{b}+1}$

C $\log_{243}(32) = \dfrac{(1-\mathrm{a})}{\mathrm{b}}$

D $\log_{40}(15) = \dfrac{\mathrm{a}+\mathrm{b}}{3-2\mathrm{a}}$

TAH-01

Q. If $\log_{10} 5 = a$ & $\log_{10} 3 = b$, then:

$$\frac{\log_5 5}{\log_5 5 + \log_5 2} = a \quad \& \quad \frac{\log_5 3}{\log_5 5 + \log_5 2} = b$$

$$\frac{1}{1 + \log_5 2} = a \qquad \boxed{\log_5 3 = b\left(1 + \frac{1-a}{a}\right) = \frac{b}{a}}$$

$$\boxed{\log_5 2 = \frac{1-a}{a}}$$

Now, i) $\log_{30} 8 = \dfrac{3\log_5 2}{(1 + \log_5 3 + \log_5 2)} = \dfrac{3\,\frac{(1-a)}{a}}{1 + \frac{b}{a} + \frac{1-a}{a}} = \boxed{\dfrac{3(1-a)}{b+1}}$

ii) $\log_{2+3} 32 = \dfrac{5\log_5 2}{5\log_5 3} = \dfrac{5\,\frac{(1-a)}{a}}{5\,\frac{b}{a}} = \boxed{\dfrac{1-a}{b}}$

iii) $\log_{40} 15 = \dfrac{\log_5 5 + \log_5 3}{\log_5 5 + 3\log_5 2} = \dfrac{1 + \frac{b}{a}}{1 + \frac{3(1-a)}{a}} = \boxed{\dfrac{a+b}{3-2a}}$

Rajkanya Anand
From bihar

IAH − 2

Q. $2\log_3(x-2)+\log_3(x-4)^2=0$

Rajkanya Anand
From Bihar

$\Rightarrow 2\log_3(x-2)+2\log_3|x-4|=0$

$\Rightarrow \log_3\big((x-2)|x-4|\big)=0$

$\Rightarrow (x-2)|x-4|=1$

Case − I > if $x-4>0$

$x>4$

$(x-2)(x-4)=1$

$x^2-6x+7=0$

$(x-3-\sqrt{2})(x-3+\sqrt{2})=0$

$x=3-\sqrt{2}$ ✗, $\boxed{3+\sqrt{2}}$ ✓

Case II > if $x-4<0$

$x<4$

$(x-2)(4-x)=1$

$x^2-6x+9=0$

$(x-3)^2=0$

$\boxed{x=3}$ ✓

QUESTION

Solve the following logarithmic equations:

1. $\log_3(x^2-3x-5)=\log_3(7-2x)$
2. $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$
3. $25^{\log_{10}x}=5+4\times\log_{10}5$
4. $1+2\log_{x+2}5=\log_5(x+2)$
5. $2\log_2(\log_2 x)+\log_{\frac{1}{2}}(\log_2(2\sqrt{2}x))=1$

Solve $25^{\log_{10}x}=5+4x^{\log_{10}5}$

TAH -03 ka 3rd Question ye hai
usmein typing error hai

TAH - 03

Q.1.> $\log_3(x^2-3x-5)=\log_3(7-2x)$

$x^2-3x-5=7-2x$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x = \underset{\times}{4}, \boxed{-3}$

Q.2.> $25^{\log_{10}x} = 5 + 4\cdot x^{\log_{10}5}$

$x^{2\log_{10}5} = 5 + 4\cdot x^{\log_{10}5}$

Put $2\cdot\log_{10}5 = t$

$x^{2t} = 5 + 4x^{t}$

again Put $x^t = a$

& $a^2 = x^{2t}$

$a^2 = 5 + 4a$

$a^2 - 4a - 5 = 0$

$(a-5)(a+1) = 0$

$a = 5, -1$

$x^t = 5, \; -1\times$

$t\log_5 x = 1$

$\log_{10}5\cdot\log_5 x = 1$

$\frac{1}{1+\log_5 2} + \log_5 x = 1$

$\log_5 x = 1 + \log_5 2$

$x = 5 * 5^{\log_5 2} = 5\cdot 2 = 10$

TAH-03 i) $\log_3(x^2-3x-5)=\log_3(7-2x)$

$x^2-3x-5=7-2x$

$x^2-3x+2x-7-5=0$

$x^2-x-12=0$

$x^2-4x+3x-12=0$

$(x-4)(x+3)=0$

$x=4,-3$

$\times$

$16-12-5=-ve \times$

$9+9-5=+ve,\ 7-6=+ve$

ii) $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$

$4^{\frac{1}{2}\log_3 3}$

$(x^2-x)^{0.5\times 2}=4^{1/2}$

$x^2-x=2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1$

$\checkmark\quad\times$

Spiral

Tah3 (3) $25^{\log_{10} x} = 5 + 4x^{\log_{10} 5}$

$5^{2\log_{10} x} = 5 + 4x^{\log_{10} 5}$

$x^{2\log_{10} 5} = 5 + 4x^{\log_{10} 5}$

let $x^{\log_{10} 5} = t$

$t^2 = 5 + 4t$

Sakshi

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5, -1$ reject bcz $N > 1$ $\log_a N$

$x^{\log_{10} 5} = 5$

$\log_x 5 = \log_{10} 5$

$\boxed{x = 10}$ Ans

iv) $1+2\log_{x+2}5 = \log_5(x+2)$

$1+\dfrac{2}{\log_5(x+2)} = \log_5(x+2)$

let, $\log_5(x+2) = t$

$1+\dfrac{2}{t} = t$

$\dfrac{t+2-t^2}{t} = 0$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$(t-2)(t+1) = 0$, $t = -1, 2$

$\Rightarrow \log_5(x+2) = -1$

$x+2 = 5^{-1}$

$x = \dfrac{1}{5} - 2 = \dfrac{1-10}{5} = \dfrac{-9}{5}$

$\Rightarrow \log_5(x+2) = 2$

$x+2 = 25$

$x = 23.$

$-\dfrac{9}{5}+2 = \dfrac{-9+10}{5}$

Q.4.) $1 + 2\log_{x+2} 5 = \log_5 (x+2)$

Put $\log_5 (x+2) = t$

$$1 + \frac{2}{t} = t$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t = 2, -1$$

$$\log_5 x+2 = 2, -1$$

$$x+2 = 25, 5^{-1}$$

$$x = 23, -\frac{9}{5}$$

Rajkanya Anand
From Bihar

5.) $2\log_2(\log_2 x) + \log_{\frac{1}{2}}(\log_2(2\sqrt{2}x)) = 1$

$\log_2(\log_2 x)^2 - \log_2(\log_2(2\sqrt{2}x)) = 1$

$$\log_2 \frac{(\log_2 x)^2}{\log_2(2\sqrt{2}x)} = 1$$

$$\frac{(\log_2 x)^2}{\log_2(2\sqrt{2}x)} = 2$$

Tah 3

$(\log_2 x)^2 = 2\cdot(\log_2 2\sqrt{2} + \log_2 x)$

$\log_2 x(\log_2 x - 2) = \cancel{2}\cdot\frac{3}{\cancel{2}} = 3$

Put $\log_2 x = t$

Rajkanya Anand
From Bihar

$t^2 - 2t - 3 = 0$

$t = 3, -1$

$\log_2 x = 3, -1$

$x = 8, \frac{1}{2}$

v) $2\log_2(\log_2 x) + \log_{\frac{1}{2}}(\log_2(2\sqrt{2}x)) = 1$

let $\log_2 x = t$

$\log_2(2\sqrt{2}x)$

$\log_2 2\sqrt{2} + \log_2 x$

$\frac{3}{2}\log_2 2 = \frac{3}{2} + \log_2 x$

$2\log_2 t - \log_2\left(\frac{3}{2} + t\right) = \log_2 2$

$\log_2\left(\frac{t^2}{\frac{3}{2}+t}\right) = \log_2 2$

$\frac{\cancel{2}t^2}{3+2t} = \cancel{2}$

$\frac{t^2}{3+2t} - 1 = 0$

$t^2 - 3 - 2t = 0$

$t^2 - 2t - 3 = 0$

$t^2 - 3t + t - 3 = 0$

$(t-3)(t+1) = 0$

$t = 3, -1$

Now,

$t = 3,$

$\log_2 x = 3$

$x = 2^3 = 8$ ✓

$t = -1,$

$\log_2 x = -1$

$x = 2^{-1} = \frac{1}{2}$ ✓

Spiral

Solution to Previous BPPs

QUESTION

Solve the following equations :

(i) $\log_{x-1} 3 = 2$

(ii) $\log_4\left(2\log_3\left(1 + \log_2(1 + 3\log_3 x)\right)\right) = \frac{1}{2}$

(iii) $\log_3\left(1 + \log_3(2^x - 7)\right) = 1$

(iv) $\log_3(3^x - 8) = 2 - x$

$3^x - 8 = 3^{2-x}$

$3^x - 8 = 3^2 \cdot 3^{-x} = \frac{9}{3^x}$

let $3^x = t$

$t - 8 = \frac{9}{t}$

(v) $\frac{\log_2(9-2^x)}{3-x} = 1$

QUESTION

Solve the following equations :

(vi) $\log_{5-x}(x^2 - 2x + 65) = 2$

(vii) $\log_{10} 5 + \log_{10}(x + 10) - 1 = \log_{10}(21x - 20) - \log_{10}(2x - 1)$

(viii) $x^{1+\log_{10} x} = 10x$

(ix) $2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$

(x) $3 + 2\log_{x+1} 3 = 2\log_3(x + 1)$

$3 + \frac{2}{\log_3(x+1)} = 2\log_3(x+1)$

$x^{1+\log_{10} x} = 10x$

$\log_{10}\left(x^{1+\log_{10} x}\right) = \log_{10}(10x) \Rightarrow (1+\log_{10} x)\cdot\log_{10} x = (1+\log_{10} x)$

let $\log_x \sqrt{5} = t$

$2t^2 - 3t + 1 = 0$

$2t^2 - 2t - t + 1 = 0$

$(2t-1)(t-1) = 0$

$t = 1/2,\ t = 1$

$\log_x \sqrt{5} = \frac{1}{2}, 1$

$\frac{1}{2}\log_x 5 = \frac{1}{2}, 1$

$\log_x 5 = 1, 2$

$x^1 = 5$ or $x^2 = 5$

$x = \pm\sqrt{5}$

$x = 5, \sqrt{5}$

Answers:

i.	$\{1+\sqrt{3}\}$	ii.	$\{3\}$	iii.	$\{4\}$	iv.	$\{2\}$
v.	$\{0\}$	vi.	$\{-5\}$	vii.	$\{3/2, 10\}$	viii.	$\{10^{-1}, 10\}$
ix.	$\{\sqrt{5}, 5\}$	x.	$\{-(3-\sqrt{3})/3, 8\}$				

1) $\log_{x-1} 3 = 2$

$3 = (x-1)^2$

$x^2+1-2x = 3$

$x^2-2x-2=0$

$\frac{2 \pm \sqrt{4-4\times(-2)}}{2}$

$\frac{2 \pm \sqrt{12}}{2}$

$1 \pm \sqrt{3}$

$1+\sqrt{3}$ ✓, $1-\sqrt{3}$ ✗

2) $\log_4(2\log_3(1+\log_2(1+3\log_3 x))) = \frac{1}{2}$

$2\log_3(1+\log_2(1+3\log_3 x)) = (4)^{1/2}$

$\cancel{2}\log_3(1+\log_2(1+3\log_3 x)) = \cancel{2}\, 1$

$1+\log_2(1+3\log_3 x) = 3$

$\log_2(1+3\log_3 x) = 2$

$1+3\log_3 x = 4$

$\cancel{3}\log_3 x = \cancel{3} \Rightarrow 1$

$\boxed{x=3}$

3) $\log_3(1+\log_3(2^x-7)) = 1$

$1+\log_3(2^x-7) = 3$

$\log_3(2^x-7) = 2$

$2^x-7 = 3^2$

$2^x-7 = 9$

$2^x = 16$

$2^x = 2^4$

$\boxed{x=4}$

4) $\log_3(3^x-8) = 2-x$

$3^x-8 = 3^{(2-x)}$

$3^x-8 = \frac{3^2}{3^x}$

Let $\Rightarrow 3^x = t$

$t-8 = \frac{9}{t}$

$t^2-8t = 9$

$t^2-8t-9=0$

$(t-9)(t+1)=0$

$t = 9, -1$

$3^x = 3^2 \Rightarrow \boxed{x=2}$ $\quad 3^x = -1$ ✗

DPP :

(i) $\log_{x-1} 3 = 2$

$(x-1)^2 = 3$

$\cancel{x^2+1-2x=3}$

$\cancel{x^2-2x-2=0}$

$x-1 = \sqrt{3}$

$x = 1 \pm \sqrt{3}$

$x = 1+\sqrt{3}$; $x = 1-\sqrt{3}$

↓ satisfy ↓ doesn't

(ii) $\log_4(2\log_3(1+\log_2(1+3\log_3 x))) = \frac{1}{2}$

$2\log_3(1+\log_2(1+3\log_3 x)) = 2$

$1+\log_2(1+3\log_3 x) = 3$

$\log_2(1+3\log_3 x) = 2$

$1+3\log_3 x = 4$

$3\log_3 x = 3$

$\boxed{x = 3}$

(iii) $\log_3(1+\log_3(2^x-7)) = 1$

$1+\log_3(2^x-7) = 3$

$\log_3(2^x-7) = 2$

$2^x - 7 = 9$

$2^x = 16$

$\boxed{x = 4}$

(iv) $\log_3(3^x-8) = 2-x$

$3^x - 8 = (3)^{2-x}$

$3^x - 8 = 3^2 \cdot 3^{-x}$

$3^x - 9\cdot 3^{-x} = 8$ let $3^x \to t$

$3^x - \frac{9}{3^x} = 8$

$t - \frac{9}{t} = 8 \Rightarrow t^2 - 9 = 8t$

$t^2 - 8t - 9 = 0$

$(t-9)(t+1) = 0$

$t = 9$; $t = -1$

$3^x = 9$; $3^x = -1$

$\boxed{x = 2}$ ↓ Not possible

(v) $\frac{\log_2(9-2^x)}{3-x} = 1$

$\log_2(9-2^x) = 3-x$

$9 - 2^x = (2)^{3-x}$

$9 - 2^x = 8\cdot 2^{-x}$

$9 - 2^x = \frac{8}{2^x}$ let $t = 2^x$

$9 - t = \frac{8}{t}$

$9t - t^2 - 8 = 0$

$t^2 - 9t + 8 = 0$

$(t-8)(t-1) = 0$

$t = 8$; $t = 1$

$2^x = 8$; $2^x = 1$

$\boxed{x = 3}$ $\boxed{x = 0}$ ✓

↳ Not possible

(5) $\dfrac{\log_2(9-2^x)}{3-x} = 1$

$\log_2(9-2^x) = 3-x$

$9-2^x = 2^{(3-x)}$

$9-2^x = \dfrac{2^3}{2^x}$

$9-2^x = \dfrac{8}{2^x}$

Let $2^x = t$

$9-t = \dfrac{8}{t}$

$9t-t^2 = 8$

$t^2-9t+8 = 0$

$(t-1)(t-8) = 0$

$t = 8, 1$

$2^x = 8$ $\quad$ $2^x = 1$

$\boxed{x=3}$ $\quad$ $2^x = 2^0$

$\times$ $\quad$ $\boxed{x=0}$

$\dfrac{\log_2(9-2^x)}{3-3 \to 0}$

N.P

(6) $\log_{5-x}(x^2-2x+65) = 2$

$x^2-2x+65 = (5-x)^2$

$\cancel{x^2}-2x+65 = 25+\cancel{x^2}-10x$

$10x-2x+65-25 = 0$

$8x+40 = 0$

$8x = -40$

$\boxed{x = -5}$

(8) $x^{1+\log_{10}x} = 10x$

~~take~~ log both side

$\log_{10}x^{1+\log_{10}x} = \log_{10}10x$

$(1+\log_{10}x)\log_{10}x = \log_{10}10+\log_{10}x$

$(1+\log_{10}x)\log_{10}x = 1+\log_{10}x$

put $\log_{10}x = t$

$(1+t)t = 1+t$

$\cancel{t}+t^2-1-\cancel{t} = 0$

$t^2-1 = 0$

$(t-1)(t+1) = 0$

$t = -1, 1$

$\log_{10}x = -1$ $\quad$ $\log_{10}x = 1$

$x = 10^{-1}$

$\boxed{x = \frac{1}{10}}$ $\quad$ $\boxed{x = 10}$

(vi) $\log_{5-x}(x^2-9x+65)=2$

$x^2-9x+65=(5-x)^2$

$x^2-9x+65=25+x^2-10x$

$8x+40=0$

$\boxed{x=-5}$

(viii) $x^{1+\log_{10}x}=10x$

taking $\log_{10}$

$(1+\log_{10}x)\log_{10}x=\log_{10}10x$

$(1+\log_{10}x)\log_{10}x=1+\log_{10}x$

let t

$(1+t)t-t=1$

$\cancel{t}+t^2-\cancel{t}=1$

$t=\pm 1$

$\log_{10}x=1$; $\log_{10}x=-1$

$x=10$; $x=\frac{1}{10}$

satisfy

(ix) $2(\log_x\sqrt{5})^2-3\log_x\sqrt{5}+1=0$

let $\log_x\sqrt{5}=t$

$2t^2-3t+1=0$

$(t-2)(t-1)=0$

$t=2$; $t=1$

$\log_x\sqrt{5}=2$; $\log_x\sqrt{5}=1$

$(x)^2=\sqrt{5}$; $x=\sqrt{5}$

$x=5$; $x=\sqrt{5}$

satisfy

Richa Thakur

(vii) $\log_{10}5+\log_{10}(x+10)-1=\log_{10}(21x-20)-\log_{10}(2x-1)$

$\Rightarrow \log_{10}5(x+10)-1=\log_{10}\frac{21x-20}{2x-1}$

$\Rightarrow \log_{10}\frac{5(x+10)}{\frac{21x-20}{2x-1}}=1$

$\Rightarrow \frac{5(x+10)(2x-1)}{21x-20}=10 \Rightarrow \frac{(x+10)(2x-1)}{21x-20}=2$

$\Rightarrow (x+10)(2x-1)=42x-40$

$\Rightarrow 2x^2-x+20x-10=42x-40$

$\Rightarrow 2x^2-23x+30=0$

$(2x-3)(x-10)=0$

$x=\frac{3}{2}$; $x=10$

Both possible.

BPP 7 $\log_{10}5 + \log_{10}(x+10) - 1 = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$\log_{10}5 + \log_{10}(x+10) - \log_{10}10 = $ "

$\log_{10}\frac{\cancel{5}(x+10)}{\cancel{10}\,2} = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$(x+10)(2x-1) = (21x-20).2$

$2x^2+19x-10 = 42x-40$

$2x^2-23x+30=0$

$2x^2-20x-3x+30=0$

$2x(x-10)-3(x-10)=0$

$x=\frac{3}{2} \checkmark \quad x=10 \checkmark$

Sakshi

BPP 8 $x^{1+\log_{10}x} = 10x$

take log both side

$(1+\log_{10}x)\log_{10}x = \log_{10}10x$

$1+\log_{10}x\,(\log_{10}x) = \log_{10}10 + \log_{10}x$

let $\log_{10}x = t$

$1+t\,(t) = 1+t$

$\cancel{1}+t^2 = 1+\cancel{t}$

sakshi

$t^2-1 = 0$

$(t+1)(t-1)=0$

$t = 1, -1$

$\log_{10}x = 1, -1$

$\log_{10}x = 1 \qquad \log_{10}x = -1$

$\boxed{x=10} \qquad \boxed{x=10^{-1}}$

2) $\log_{10} 5 + \log_{10}(x+10) - 1 = \log_{10}(21x-20) - \log_{10}(2x-1$

$\log_{10}(5(x+10)) - 1 = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$\log_{10}(5(x+10)) - \log_{10}\left(\frac{21x-20}{2x-1}\right) = 1$

$\log_{10}\left(\frac{5(x+10)(2x-1)}{21x-20}\right) = 1$

$\frac{5(x+10)(2x-1)}{21x-20} = 10$

$(x+10)(2x-1) = 42x-40$

$2x^2 - x + 20x - 10 - 42x + 40 = 0$

$2x^2 - 43x + 20x + 30 = 0$

$2x^2 - 23x + 30 = 0$

$2x^2 - 20x - 3x + 30 = 0$

$2x(x-10) - 3(x-10) = 0$

$(2x-3)(x-10) = 0$

$x = 10, \frac{3}{2}$.

3) $2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$

$\log_x \sqrt{5} = t$

$2t^2 - 3t + 1 = 0$

$2t^2 - 2t - t + 1 = 0$

$2t(t-1) - 1(t-1) = 0$

$(2t-1)(t-1) = 0$

$t = 1, \frac{1}{2}$

$\log_x \sqrt{5} = 1$

$\cancel{\sqrt{5} = x^1}$

$\sqrt{5} = x^1$

$\boxed{x = \sqrt{5}}$

$\log_x \sqrt{5} =$

$\sqrt{5} = x$

$\boxed{x = 5}$

BPP9 $2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$

let $\log_x \sqrt{5} = t$

$2t^2 - 3t + 1 = 0$

$(2t-1)(t-1) = 0$

sakshi

$t = 1, 1/2$

$\log_x \sqrt{5} = 1/2$ $\qquad$ $\log_x \sqrt{5} = 1$

$\sqrt{5} = x^{1/2}$ $\qquad$ $\boxed{\sqrt{5} = x}$

$\sqrt{5} = \sqrt{x}$

$\boxed{x = 5}$

(10) $3 + 2\log_{x+1} 3 = 2\log_3 (x+1)$

$$3 + \frac{2}{\log_3 x+1} = 2\log_3 (x+1)$$

Let $\log_3 x+1 = t$

$$3 + \frac{2}{t} = 2t$$

$$3t + 2 = 2t^2$$

$$2t^2 - 3t - 2 = 0$$

$$2t^2 - 4t + t - 2 = 0$$

$$2t(t-2) + 1(t-2) = 0$$

$$(2t+1)(t-2) = 0$$

$$t = 2, -\frac{1}{2}$$

$\log_3 x+1 = 2$

$\cancel{x+1 = 2^3}$

$\cancel{x+1 = 8}$

$\cancel{\boxed{x = 7}}$

$x + 1 = 3^2$

$x + 1 = 9$

$\boxed{x = 8}$

$\log_3 x+1 = -\frac{1}{2}$

$x + 1 = 3^{-1/2}$

$x + 1 = \frac{1}{\sqrt{3}}$

$x = \frac{1}{\sqrt{3}} - 1$

$\boxed{x = \frac{1-\sqrt{3}}{\sqrt{3}}}$

(x) $3 + 2\log_{x+1} 3 = 2\log_3(x+1)$

$3 + \frac{2}{\log_3(x+1)} = 2\log_3(x+1)$

Let $\log_3(x+1) \to t$

$3 + \frac{2}{t} = 2t$

$3t + 2 - 2t^2 = 0$

$2t^2 - 3t - 2 = 0$

$(2t+1)(t-2) = 0$

$t = \frac{-1}{2}$; $t = 2$

$\log_3(x+1) = \frac{-1}{2}$

$(3)^{-1/2} = x+1$

$\frac{1}{\sqrt{3}} = x+1$

$x = \frac{1}{\sqrt{3}} - 1$

$x = \frac{1-\sqrt{3}}{\sqrt{3}}$ possible

$x = -\left(\frac{3-\sqrt{3}}{3}\right)$ A

$x = 8$

$\log_3(x+1) = 2$

$x + 1 = 9$

$x = 8$

possible

Mathematical Gyaan

⭐ Square of an integer leaves the remainder zero or one upon division by 4 that is every square is of the form 4K or 4K+1

Any Integer is of type $2m$ or $2m+1$

$(2m)^2 = 4m^2 = 4k$ - Form

$(2m+1)^2 = 4m^2+4m+1$

$= 4(m^2+m)+1$

$= 4k+1$ - Form.

$16249\underline{\underline{99}}$ - form $4k+3$

Not a perfect square

$$\begin{array}{r} 24 \\ 4\,\overline{)\,99} \\ \underline{8} \\ 19 \\ \underline{16} \\ \underline{3} \end{array}$$

Question

Prove that no integer in the following sequence is a perfect square:

$$11, 111, 1111, 11111, \ldots\ldots\ldots$$

each no: in sequence leaves remainder 3 on dividing by 4 hence none of the term in seq. can be a perfect square

THANK YOU